Semester	JAN 2022
Open to semester	8,12,14
Course code	**MTH411**
Course title	**Functional Analysis**
Credits	4 /
Course Coordinator & participating faculty (if any)	Chandrasheel Bhagwat
Nature of Course	Lectures
Pre-requisites	Linear algebra, Calculus, Real Analysis, Complex Analysis (up to the statements of basic theorems about holomorphic maps), Measure theory (up to L^p-spaces over real and complex numbers
Objectives (goals, type of students for whom useful, outcome etc)	In this course, a student will learn the analytic and topological aspects of various function spaces and operators on them, the geometry of Hilbert spaces, and analogs of theorems in classical Fourier Analysis. In the last part of the course, a student would gain a fair understanding of the spectral structure of hermitian and normal operators on Hilbert spaces through their spectrum and the language of C*-algebras. At the end of this course, a student would have seen the following two themes in functional analysis in reasonably great detail (this course stands out from a typical Functional analysis course at other places in this regard.) a) The main theorems on Banach spaces in a unified manner using semi-norms, b) Spectral theorem for normal bounded operators on separable Hilbert spaces. Utility: This course is useful for all mathematics students, Essential for students who want to study subjects like Harmonic Analysis, Complex Analysis, Partial Differential Equations, Probability, Representation theory.
Course contents (details of topics /sections with no. of lectures for each)	Normed linear spaces: Examples, bounded linear maps between normed linear spaces, Subspaces and quotients, Locally compact normed linear spaces, Linear functionals, and Hahn-Banach theorem (~10 Lectures)

	Banach spaces: Completeness for normed linear spaces, Examples, closed subspaces and quotients, examples, Seminorms, Zabreiko's lemma, Bounded inverse theorem, Open mapping theorem, Closed graph theorem, Uniform boundedness principle, Dual spaces, Weak and weak* topologies, Banach-Alaoglu theorem (~10 Lectures) Hilbert spaces: Hermitian inner products, Geometry of Hilbert spaces, Orthogonal projections, Riesz representation theorem for the dual of a Hilbert space, Orthonormal basis, Fourier expansion, Compact operators on Hilbert spaces (~10 Lectures) C*-algebras: Spectral theory, Gelfand-Naimark theorem for abelian C*-algebras, Spectrum of a bounded linear map, Spectral theorem of bounded normal operators on Hilbert spaces in form of spectral measures (~10 Lectures)
Evaluation /assessment	End-Sem Examination-50% Mid-Sem Examination-50% Others-%
Suggested readings (with full list of authors, publisher, year, edn etc.)	The main reference is my typed notes on functional analysis. During the course, I will share the notes regularly. They may differ from the previous version of this course from the January 2021 semester. Some of the other useful references (they treat the subject differently and also cover more material): 1. Course in functional analysis by John B. Conway, Springer GTM. 2. Introduction to Hilbert space and the theory of spectral multiplicity by P. Halmos, Chelsea Publishing, New York. 3. Functional Analysis by Peter D. Lax, Wiley Interscience publishers.